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## PARAMETRIC IDENTIFICATION OF LINEAR DYNAMIC NEIGHBORHOOD MODELS WITH VARIABLE NEIGHBORHOODS

Sedykh Irina Aleksandrovna<sup>a</sup>, Anatoly Mikhailovich Shmyrin<sup>a\*</sup>

<sup>a</sup> Department of Higher Mathematics, Lipetsk State Technical University, Lipetsk, RUSSIA.

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### ABSTRACT

The paper presents the definition of the dynamic input-state neighborhood model, the structure of which is determined by the set of nodes and the neighborhood links between model nodes according to control actions and states. The dynamic neighborhood model functions in discrete time. The states of each node in the next point in time depend on the control actions and the states of adjacent nodes and change under the influence of the state recalculation function. The paper considers a neighborhood model with variable neighborhoods, which is distinguished by dynamic links between the nodes of the system, changing at each discrete point in time of its operation. In this model, the entire set of neighborhood links is divided into intersecting subsets that form layers. For dynamic neighborhood models, the problem of parametric identification is given, the identification criterion is shown. The method is considered and the block diagram of the solution of the parametric identification problem for linear neighborhood models in matrix form is given.

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## 1. INTRODUCTION

Neighborhood models are notable for their flexibility in describing the neighborhood connections of the system and are used to model complex spatially distributed objects. The simplest classes of neighborhood models include the linear symmetric neighborhood model “input-state” and the mixed model “input-state-output” (Bljumin et al, 2005; Bljumin et al, 2006; Shmyrin et al, 2015; Shmyrin et al, 2016) . Dynamic neighborhood models make it possible to take into account not only neighborhood relationships between nodes of a distributed system or process, but also their development over time (Shmyrin et al, 2014; Shmyrin et al, 2018).

Nondeterministic models belong to one of the classes of dynamic neighborhood models, the relations between nodes in which are specified in layers. When such models operate at each discrete point in time, one or more layers are selected for launch (Bljumin et al, 2010; Shmyrin et al, 2017).

There is another kind of non-deterministic neighborhood models, but they use agents that move in non-deterministic neighborhoods and interact with each other (Shang, 2010; Shang, 2017).

Examples of the use of neighborhood systems are the bilinear relational neighborhood model of the diffusion stage of sugar production (Shmyrin et al, 2018), hierarchical neighborhood models for the ventilation system (Shmyrin et al, 2018), the linear and bilinear neighborhood model of a wastewater treatment plant, a dynamic non-deterministic neighborhood model of operation cement production, the surrounding model of the process of forming the temperature of the hot-rolled strip coiling and others.

In this paper, we consider the dynamic input-state neighborhood models with variable neighborhoods (Sedykh, 2018), which generalize the neighborhood models introduced earlier and differ in the possibility of changing connections between the nodes of the system during its operation. These neighborhood models allow you to model complex dynamic spatially distributed systems and processes with a time-varying structure. For the considered neighborhood models, we present the formulation of the problem and the criterion for parametric identification. We show the method of solving the problem of parametric identification. Parametric identification of models with variable neighborhoods is carried out for each variable neighborhood separately.

## 2. DYNAMIC VILLAGE MODELS WITH VARIABLE ENVIRONMENTS

Dynamic neighborhood model "input-state" with constant neighborhoods is described by a set  $NS^G = (N, X, V, G, X[0])$ , where:

- 1)  $N$  – structure of the neighborhood model;
- 2)  $X \in R^{\sum_{i=1}^n p_i}$  – block vector of states of the neighborhood model at the current time;
- 3)  $V \in R^{\sum_{i=1}^n m_i}$  – block vector of control actions of the neighborhood model at the current time;
- 4)  $G = (G_1 \dots G_n)^T$  – vector function of recalculation of states of the neighborhood model;
- 5)  $X[0]$  – initial state of the model.

The structure of the neighborhood model is defined by a pair of  $N = (A, O)$ , where  $O = O_x \cup O_v$ ;  $O_x = \bigcup_{i=1}^n O_x[i]$  и  $O_v = \bigcup_{i=1}^n O_v[i]$  – neighborhoods of node connections by states and control actions.

The time in the model is a discrete value in increments  $\Delta t = 1$ , initial time of operation  $t_0 = 0$ . Under the influence of the state recalculation function  $G_i$  knot  $a_i$  next time  $t+1$  goes into state  $X[t+1, i] \in R^{p_i}$ .

In the considered dynamic neighborhood model, the parameters are constant, and the connections

between the nodes within the neighborhoods do not depend on time. The neighborhood model with variable neighborhoods is distinguished by dynamic links between the nodes of the system, changing at each discrete time of operation.

Let there be  $L$  possible combinations of neighborhood connections between system nodes or layers  $O = \{O^1, O^2, \dots, O^L\}$ , where  $O^p = O_x^p \cup O_v^p$  ( $p=1, \dots, L$ ). In each  $p$ -th layer includes all nodes of the neighborhood model  $A = \{a_1, a_2, \dots, a_n\}$  and a subset of the connections between them. We introduce the notation:  $X_i^p[t] = \left( X^T[t, j_1^p], \dots, X^T[t, j_r^p] \right)^T$ ,  $V_i^p[t] = \left( V^T[t, k_1^p], \dots, V^T[t, k_h^p] \right)^T$ , where for all  $s=1, \dots, r$ , with node  $a_{j_s}^p \in O_x^p[i]$  – node in the neighborhood of the node  $a_i$  as in the layer  $O^p$ ;  $X[t, j_s^p]$  – state in node  $a_{j_s}^p$  at time  $t$ ; for all  $w=1, \dots, h$  knot  $a_{k_w}^p \in O_v^p[i]$  – neighborhood node  $a_i$  on the control action in the layer  $O^p$ ;  $V[t, k_w^p]$  – control action on the node  $a_{k_w}^p$  at time  $t$ ;  $i=1, \dots, n$ . For each node  $a_i \in A$  and  $p$ -th layer  $O^p$  define the functions of recalculating the states  $G_i^p$ :

$$X[t+1, i] = G_i^p \left( X_i^p[t], V_i^p[t] \right) = G_i^p[t]. \quad (1)$$

Let a vector function be given  $P: T \rightarrow \{0,1\}^L$  active layer selection at each discrete point in time  $t \in T = \{t_0, t_0 + 1, \dots\}$ , depending in general on the current state of the model and the current control action. Layer  $O^p$  is active at time  $t$ , if the specified activation condition is met for it  $P_p[t] = 1$ . At each current time in the neighborhood model with variable neighborhoods only one layer can be active, according to the equations of which the model states are recalculated. Functions  $G_i^p$  in (1) will also depend on the number of the currently active layer:

$$X[t+1, i] = G_i(p[t], X_i^{p[t]}[t], V_i^{p[t]}[t]) = G_i^p[t]. \quad (2)$$

In the linear case for each layer  $O^p$  the equation (2) has the appearance:

$$\begin{aligned} X[t+1, i] = & g_{0, \dots, 0, 0}^{p[t]}[i] + g_{1, \dots, 0, 0}^{p[t]}[i] \cdot X[t, j_1] + \dots + g_{0, \dots, 1, 0}^{p[t]}[i] \cdot X[t, j_r] + \\ & + g_{0, \dots, 0, 1}^{p[t]}[i] \cdot V[t, k_1] + \dots + g_{0, \dots, 0, 1}^{p[t]}[i] \cdot V[t, k_h], \end{aligned} \quad (3)$$

where  $g_{c_1, \dots, c_r, q_1, \dots, q_h}^{p[t]}[i]$  – matrix model parameter for the node  $a_i$  by states.

### 3. PARAMETRIC IDENTIFICATION OF NEIGHBORHOOD MODELS WITH VARIABLE NEIGHBORHOODS

Consider the solution of the parametric identification problem for a neighborhood model with

variable neighborhoods. Let for each node  $a_i \in A$  and each layer  $O^p \in O$  ( $p=1, \dots, L$ ) set learning set  $\hat{H}^{ip} = \{\hat{H}_1^{ip}, \dots, \hat{H}_{M_p}^{ip}\}$ , consisting of  $M_p$  source data tuples:

$$\hat{H}_m^{ip} = (X_m[t, i_1^p], \dots, X_m[t, i_r^p], V_m[t, k_1^p], \dots, V_m[t, k_h^p], X_m^p[t+1, i]), \quad m=1, \dots, M_p.$$

To solve the problem, it is necessary to find the model parameters for each node  $a_i$  ( $i=1, \dots, n$ ) and each layer  $O^p$ , meeting the criterion of parametric identification

$$E_i^p(G_i^p) = \sum_{m=1}^{M_p} \left\| X_m^p[t+1, i] - G_i^p \left( X_m[t, i_1^p], \dots, X_m[t, i_r^p], V_m[t, k_1^p], \dots, V_m[t, k_h^p] \right) \right\|^2 \rightarrow \min \quad (4).$$

Parametric identification of models with variable neighborhoods is carried out for each variable neighborhood separately.

Consider the identification of the linear neighborhood model in more detail (3). We introduce the notation of block matrices for an arbitrary knot  $a_i \in A$ :

$$A_i^p = \begin{bmatrix} 1 & (X_1[t, i_1^p])^T & \dots & (X_1[t, i_r^p])^T & (V_1[t, k_1^p])^T & \dots & (V_1[t, k_h^p])^T \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & (X_M[t, i_1^p])^T & \dots & (X_M[t, i_r^p])^T & (V_M[t, k_1^p])^T & \dots & (V_M[t, k_h^p])^T \end{bmatrix} \quad (5);$$

$$L_i^p = \begin{bmatrix} g_{0,0,\dots,0}^p[i] & g_{1,0,\dots,0}^p[i] & \dots & g_{0,0,\dots,1}^p[i] & g_{0,0,\dots,0}^p[i] & \dots & g_{0,0,\dots,0}^p[i] \\ 0,0,\dots,0 & 0,0,\dots,0 & & 0,0,\dots,0 & 1,0,\dots,0 & & 0,0,\dots,1 \end{bmatrix}^T \quad (6);$$

$$B_i^p = \begin{bmatrix} X_1^p[t+1, i] & \dots & X_M^p[t+1, i] \end{bmatrix}^T \quad (7).$$

In the matrix  $A_i^p$   $V_m[t, k_w]$ ,  $X_m[t, j_s]$  – node control values  $a_{k_w}$  and node states  $a_{j_s}$  respectively at some current time  $t$  tuple  $m$  ( $m=1, \dots, M$ ).

In the matrix  $L_i^p$   $g_{c_1, \dots, c_r, [i]}$   $q_1, \dots, q_h$  – unknown matrix parameter for the node  $a_i$ . In the matrix  $B_i^p$   $X_m^p[t+1, i]$  – state values at time  $t+1$  node  $a_i$  tuple  $m$ .

Parametric identification criterion (4) in matrix form has the form:

$$E_i^p(G_i^p) = \left\| B_i^p - A_i^p \cdot L_i^p \right\|^2 \rightarrow \min. \quad (8)$$

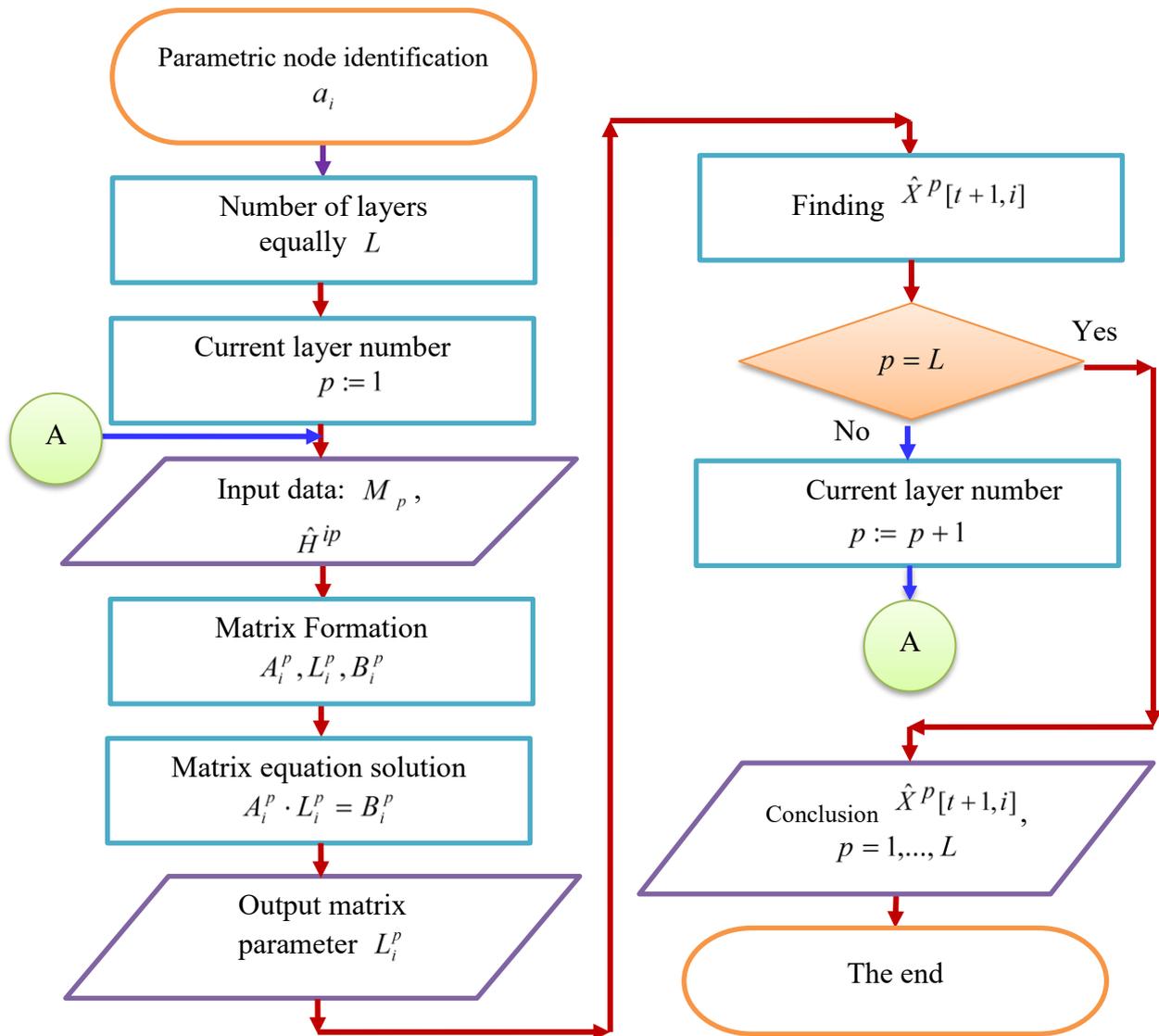
In matrix form, the search for a solution to the problem of parametric identification of a system (3), meeting the criteria (8), is to solve a system of overdetermined matrix equations (9) and finding

the matrix  $L_i^p$ :

$$A_i^p \cdot L_i^p = B_i^p \quad (9),$$

as a solution to a problem that satisfies (8), can choose normal pseudo solutions of matrix equations:

$L_i^p = (A_i^p)^+ \cdot B_i^p$ , where  $(A_i^p)^+$  – pseudoinverse to  $A_i^p$ . Thus, for the parametric identification of the entire linear dynamic neighborhood model, it is necessary to find  $n \cdot L$  matrices  $L_i^p$ , satisfying (8).



**Flowchart 1:** Block diagram of the parametric identification method node  $a_i$  linear dynamic neighborhood model

After finding matrices with parameters  $L_i^p$ , can find model values of states  $\hat{X}^p[t+1, i]$  according to the formula:  $\hat{X}^p[t+1, i] = A_i^p \cdot L_i^p$ .

The standard error of identification of neighborhood models is calculated by

$$E = \frac{1}{nML} \sum_{i=1}^n \sum_{p=1}^L \sum_{m=1}^{M_p} \left\| X_m^p[t+1, i] - \hat{X}_m^p[t+1, i] \right\|^2 \quad (10).$$

Block diagram of the method of parametric identification of the node  $a_i$  linear dynamic neighborhood model is shown in Flowchart 1.

After parametric identification, it is possible to study the properties of the obtained neighborhood model.

## 4. CONCLUSION

The paper provides a brief overview of the history of the development of neighborhood modeling, the main classes of neighborhood models. The dynamic input-state neighborhood models with variable neighborhoods are considered, the distinctive features of which are neighborhood connections between nodes that change during the operation of the system. For neighborhood models, the formulation of the parametric identification problem is given and a method for solving it is proposed for linear functions of state recalculation. In the case of polynomial functions, parametric identification is performed similarly. Using the above dynamic neighborhood models, one can model, predict and investigate the change of states of complex dynamic spatially distributed systems and processes with a variable time structure.

## 5. DATA AND MATERIAL AVAILABILITY

All information on this study can be requested to the corresponding author.

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Irina A. Sedykh is an Associate Professor at the Department of Higher Mathematics of Lipetsk State Technical University. Sedykh candidate of physical and mathematical sciences. He has monographs and articles in the field of neighborhood modeling of dynamic systems, neural networks and Petri nets.



Professor Dr. Anatoly M. Shmyrin is Professor and Head of the Department of Higher Mathematics of Lipetsk State Technical University. Professor Shmyrin holds a Doctor of Technical Sciences degree. He does research in the fields of neighborhood modeling of dynamic systems, neural networks and Petri nets.